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Interaction of the nonrelativistic electrons in the pulsed field of two laser waves

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Abstract. Interaction of two non-relativistic classical electrons in the presence of two strong pulsed laser waves of various frequencies, pulse durations and intensities is theoretically studied. It is shown, that there is a range of wave intensities in which average effective force of the electrons interaction becomes negative in current of the certain interval of time.

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1 Introduction

The study of electron interaction in the presence of the field of a plane electromagnetic wave dates back to the 1960s. In 1967, Oleinik pointed out opportunity of formation coupled electrons at presence of an external electromagnetic wave [1]. The processes of resonant, nonresonance scattering of an electron by a positron and other similar processes in the field of a light wave were wide examined by the Roshchupkin's scientific group [2–9] and other scientists [10,11]. In the middle of 1980, Sokolov and Kazantsev [12] analytically studied the interaction of two classical relativistic electrons in the presence of the field of a circularly polarized wave of arbitrary intensity. They demonstrated the possibility of sign reversal of the interaction potential for relativistic electrons in the presence of relatively strong fields. In 1989, Zavtrak [13] noticed opportunity of formation by electrons of the connected conditions at influence of an external electromagnetic field. In the previous works of authors it was detected the reversal of sign of average effective force of the non-relativistic electrons interaction [14] and ions interaction in a crossover region [15] in the presence of one strong pulse laser wave.

In this paper we study the interaction of the two electrons in the presence of two strong pulsed laser waves propagating in the opposite directions. The problem is considered outside the framework of the dipole approximation (with an accuracy of about v/c). It is shown, that effective interaction force between electrons under the influence of pulsed laser field can fundamentally differ from the Coulomb law and become an attractive force.

2 The effective interaction force

Let's investigate the two electrons interaction in the presence of two strong pulsed laser waves extending towards each other along the axis Z. We assume that the electric strengths can be represented as:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2,$$

$$\mathbf{E}_1 = E_{01} \exp\left(-\frac{t^2}{t_1^2}\right) \cos\left(\omega_1 \xi_-\right) \mathbf{e}_x, \quad \xi_\pm = t \pm \frac{z}{c}, \tag{1}$$

$$\mathbf{E}_2 = E_{02} \exp\left(-\frac{t^2}{t_2^2}\right) \cos\left(\omega_2 \xi_+\right) \left(\mathbf{e}_x \cos\varphi + \mathbf{e}_y \sin\varphi\right). \tag{2}$$

Here E_{0j} and ω_j $(j = 1, 2)$ are the electric field strengths at the laser pulse peak and the frequencies of the waves, respectively; $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors directed along the x and y axis; t_j is the laser pulse durations of first and second waves $(j = 1, 2)$; φ is the angle between electric field strengths $\mathbf{E}_1$ and $\mathbf{E}_2$.

We assume that the characteristic oscillation time $(\sim\omega_j^{-1})$, is significantly less than the laser pulse duration, so that the following condition is satisfied:

$$\tau_j = \omega_j t_j \gg 1, \quad j = 1, 2. \tag{3}$$

The motion of the classical non-relativistic electrons in the presence of the pulsed laser field (1)–(2) is represented by Newton equations accurate within to terms of the order of $v/c \ll 1$:

$$m\ddot{\mathbf{r}}_1 = -e\left(\mathbf{E}(t, r_1) + \frac{1}{c}\dot{\mathbf{r}}_1 \times \mathbf{H}(t, r_1)\right) - \frac{e^2(\mathbf{r}_2 - \mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|^3},$$

$$m\ddot{\mathbf{r}}_2 = -e\left(\mathbf{E}(t, r_2) + \frac{1}{c}\dot{\mathbf{r}}_2 \times \mathbf{H}(t, r_2)\right) + \frac{e^2(\mathbf{r}_2 - \mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|^3}. \tag{4}$$

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Here e and m are the absolute values of the electron charge and the electron mass, respectively; c is the velocity of light in free space; $\mathbf{r}_1$ and $\mathbf{r}_2$ are the radius vectors of the first and the second electron, respectively; $\mathbf{E}(t, r_j)$ is the electric field strength (see Eqs. (1, 2)). $\mathbf{H}(t, r_j)$ is the magnetic field strength given by:

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2,$$
$$\mathbf{H}_1 = H_{01} \exp\left(-\frac{t^2}{t_1^2}\right) \cos(\omega_1 \xi_-) \mathbf{e}_y, \quad \xi_\pm = t \pm \frac{z}{c}, \tag{5}$$
$$\mathbf{H}_2 = H_{02} \exp\left(-\frac{t^2}{t_2^2}\right) \cos(\omega_2 \xi_+) (\mathbf{e}_x \sin\varphi - \mathbf{e}_y \cos\varphi). \tag{6}$$

Here H_{0j} $(j = 1, 2)$ is the magnetic field strength at the laser pulse peak.

Let us transfer in the center-of-mass system:

$$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1, \quad \mathbf{R} = \frac{1}{2}(\mathbf{r}_2 + \mathbf{r}_1). \tag{7}$$

Then, the system of equations (4) is represented as:

$$\begin{cases} \dfrac{d^2\xi}{d\tau^2} = \mathbf{F}, \\ \dfrac{d^2\Xi}{d\tau^2} = \dfrac{1}{2}\mathbf{K}. \end{cases} \tag{8}$$

Here $\mathbf{F}$ is the effective force vector (in $\mu c\omega$ units) determining the variation in the relative distance between the electrons:

$$\mathbf{F} = [\mathbf{e}_x F_x + \mathbf{e}_y F_y + \mathbf{e}_z F_z], \tag{9}$$

$$\begin{cases} F_x = -b_- + \dot{\Xi}_z b_+ + \frac{1}{2}\dot{\xi}_z a_- + \sqrt{\beta_0}\beta_1 \frac{\xi_x}{\xi^3}, \\ F_y = +\left(1 + \dot{\Xi}_z\right) b_2 - \frac{1}{2}\dot{\xi}_z a_2 + \sqrt{\beta_0}\beta_1 \frac{\xi_y}{\xi^3}, \\ F_z = -\dot{\Xi}_x b_+ - \dot{\Xi}_y b_2 - \frac{1}{2}\dot{\xi}_x a_- + \frac{1}{2}\dot{\xi}_y a_2 \\ \quad +\sqrt{\beta_0}\beta_1 \frac{\xi_z}{\xi^3}, \end{cases} \tag{10}$$

and $\mathbf{K}$ is the force vector (in $\mu c\omega$ units) determining the variation in the distance of the center-of-mass:

$$\begin{cases} K_x = -a_+ + \dot{\Xi}_z a_- + \frac{1}{2}\dot{\xi}_z b_+, \\ K_y = -\left(1 + \dot{\Xi}_z\right) a_2 + \frac{1}{2}\dot{\xi}_z b_2, \\ K_z = -\dot{\Xi}_x a_- - \frac{1}{2}\dot{\xi}_x b_+ + \dot{\Xi}_y a_2 + \frac{1}{2}\dot{\xi}_y b_2, \end{cases} \tag{11}$$

where:

$$\begin{cases} a_\pm = \frac{1}{\sqrt{\beta_0}}\eta_1 f_1 C_1 \pm \sqrt{\beta_0}\eta_2 f_2 C_2 \cos(\varphi), \\ b_\pm = \frac{1}{\sqrt{\beta_0}}\eta_1 f_1 D_1 \pm \sqrt{\beta_0}\eta_2 f_2 D_2 \cos(\varphi), \end{cases} \tag{12}$$

$$f_j = \exp\left(-\frac{\tau^2}{\tau_j^2}\right), \qquad \begin{cases} a_2 = \sqrt{\beta_0}\eta_2 C_2 \sin(\varphi), \\ b_2 = \sqrt{\beta_0}\eta_2 D_2 \sin(\varphi), \end{cases} \tag{13}$$

$$\begin{cases} C_1 = \cos\left(\frac{1}{\sqrt{\beta_0}}(\tau - \Xi_z)\right) \cos\left(\frac{1}{\sqrt{\beta_0}}\left(\frac{\xi_z}{2}\right)\right), \\ C_2 = \cos\left(\frac{1}{\sqrt{\beta_0}}(\tau + \Xi_z)\right) \cos\left(\frac{1}{\sqrt{\beta_0}}\left(\frac{\xi_z}{2}\right)\right), \end{cases} \tag{14}$$

$$\begin{cases} D_1 = \sin\left(\frac{1}{\sqrt{\beta_0}}(\tau - \Xi_z)\right) \sin\left(\frac{1}{\sqrt{\beta_0}}\left(\frac{\xi_z}{2}\right)\right), \\ D_2 = \sin\left(\frac{1}{\sqrt{\beta_0}}(\tau + \Xi_z)\right) \sin\left(\frac{1}{\sqrt{\beta_0}}\left(\frac{\xi_z}{2}\right)\right). \end{cases} \tag{15}$$

In expressions (8)–(15) we use the following notations: $\tau = \sqrt{\omega_1\omega_2}t$, $\xi = \mathbf{r}/\sqrt{\lambda_1\lambda_2}$, $\mathbf{\Xi} = \mathbf{R}/\sqrt{\lambda_1\lambda_2}$. Parameters η_j $(j = 1, 2)$ and β_0, β_1 are written as:

$$\eta_j = v_j/c = eE_{0j}\lambda_j/\mu c^2 \ll 1, \qquad \beta_0 = \omega_2/\omega_1,$$
$$\beta_1 = e^2/\mu c^2\sqrt{\lambda_1\lambda_2} \ll 1, \qquad \mu = m/2. \tag{16}$$

Here η_j are the relativistic invariant parameters equal to the ratio of the field work (at the peak of the laser pulse) at a wavelength to the rest energy of an electron with reduced mass μ. v_j is the velocity of the oscillatory motion of an electron with reduced mass at the laser pulse peak first or second wave, and β_1 is a parameter equal to the ratio of the Coulomb interaction energy of electrons at a wave-length to the rest energy of an electron with reduced mass. Below, we assume that:

$$v_j \ll c, \quad j = 1, 2. \tag{17}$$

Stress that in expression (9) for the effective force, the term related to the laser pulse is of the order of v/c $(\xi_z = kr_z \sim v/c \ll 1)$ so that it vanishes in the dipole approximation when $(k = 0,\ H = 0)$.

With allowance for expression (9) one can easily obtain the projection of the effective force along the relative distance between the electrons:

$$F_\xi = \mathbf{F} \cdot \mathbf{e}_\xi = \frac{1}{\xi}[\xi_x F_x + \xi_y F_y + \xi_z F_z] + \sqrt{\beta_0}\beta_1 \frac{1}{\xi^2}. \tag{18}$$

Note that if $F_\xi > 0$ $(F_\xi < 0)$ the repulsion (attraction) of electrons takes place. The averaging of expression (18), and the relative distance between the electrons with respect to the period of fast oscillations yields the following results:

$$\bar{F}_\xi = \frac{1}{2\pi}\int_0^{2\pi} F_\xi d\tau, \qquad \bar{\xi} = \frac{1}{2\pi}\int_0^{2\pi} \xi d\tau. \tag{19}$$

The system of equations (8, 18, 19) is solved numerically.

We assume that in the initial state, the electrons are located on the X-axis (see Fig. 1). The initial distance between electrons is equal $\xi_0 = 1.8 \times 10^{-1}$ μm, the electrons possess an energy of about 3 eV.

The electrons moves towards each other along an axis X. Time for which electrons approach on the minimal distance is equal $\tau = 1.8 \times 10^{-13}$ s. The frequencies $(\omega_1,\ \omega_2)$, pulse durations $(t_1,\ t_2)$ and wave intensities $(\eta_1,\ \eta_2)$ are varied. On all figures the dashed line corresponds to interaction of electrons without influence of an external field.

3 Case of identical wave frequencies

Figures 2–4 show the average effective force of the electrons interaction as function of the dimensionless time at assumption that the frequencies are equal $\omega_1 = \omega_2 = 3.3 \times 10^{15}\,\mathrm{s}^{-1}$. It is found that qualitative difference of

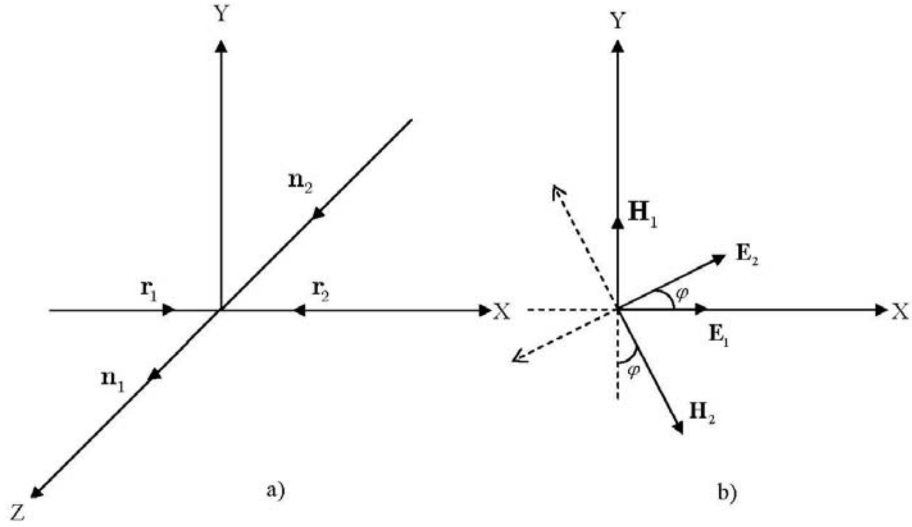

Fig. 1. Interaction geometry of two classical electrons in the presence of two strong pulsed laser waves which propagating in the opposite directions. Here $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ are initial radius-vectors of electrons; $\boldsymbol{n}_1$, $\boldsymbol{n}_2$ are vectors of the wave direction's propagation; $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{H}_1$, $\mathbf{H}_2$ are the electric and magnetic field strengths at the laser pulse peak (see Eqs. (1, 2, 5, 6)).

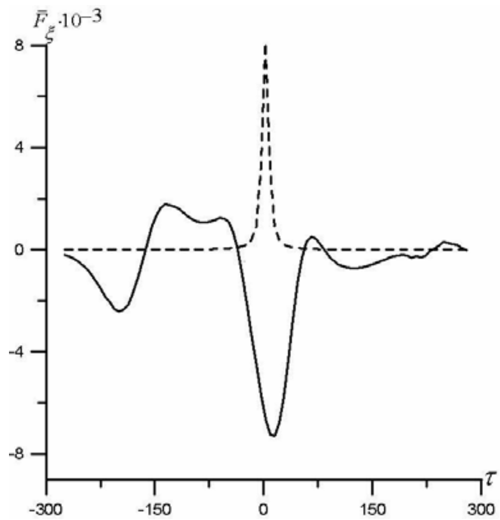

Fig. 2. The average effective force of the electrons interaction in units of the Coulomb energy at the wave-length vs. the dimensionless time (full line) (see Eqs. (18, 19)). The laser field intensity of the first and second waves are equal $I_1 = 0.35\times10^{15}$ and $I_2 = 3.16 \times 10^{15}$ W/cm^2, respectively. The pulse durations are equal $t_1 = t_2 = 1.8 \times 10^{-13}$ s. The minimum distance between electrons is $\xi_{\min} = 1.57 \times 10^{-2}\,\mu$m.

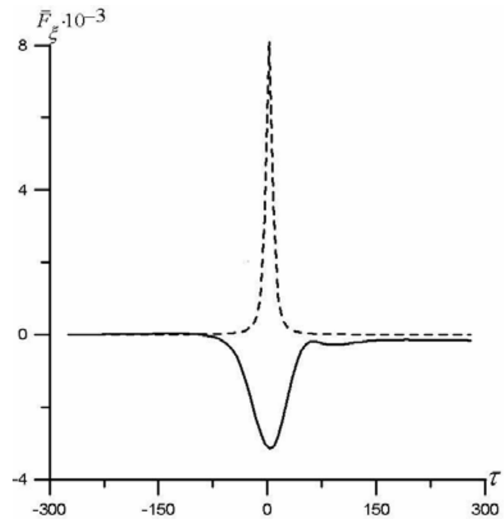

Fig. 3. The average effective force of the electrons interaction in units of the Coulomb energy at the wave-length vs. the dimensionless time (full line) (see Eqs. (18, 19)). The laser field intensity of the first and second waves are equal $I_1 = 0.35 \times 10^{15}$ and $I_2 = 3.16 \times 10^{15}$ W/cm^2. The pulse durations are $t_1 = 1.8 \times 10^{-13}$ s, $t_2 = 1.5 \times 10^{-13}$ s. The minimum distance between electrons is $\xi_{\min} = 2.07 \times 10^{-2}\,\mu$m.

average effective force of the electrons interaction from the Coulomb law falls on area of intensities: $I_{1,2} \in [0.35...3.16]\times10^{15}$ W/cm^2. One can see the well observable area of the negative values of force (the area of an electrons attraction), which exists approximately 0.26×10^{-13} s. Let's note that for the given case the wave intensities should differ from each other or influence of a field will disappear.

In Figure 3 the pulse durations are different. Intensities at which the area of negative values of force is observed are just the same. However magnitude of force becomes smaller.

In Figure 4 one can see, that the kind of force has essentially changed. It is obvious, that at such initial conditions the contribution only that wave is essential, whose pulse duration considerably exceeds pulse duration of other wave. It is natural to assume, that the given result is reduced to a problem in a field of one wave (4) and does not represent interest in this work.

4 Case of various wave frequencies

Figures 5–9 shows the average effective force of the electrons interaction versus the dimensionless time for a case when frequencies are differed. In Figures 5, 6 one can see, that the stable area of negative values of forces collapses (it takes place when the frequencies differ to little degree).

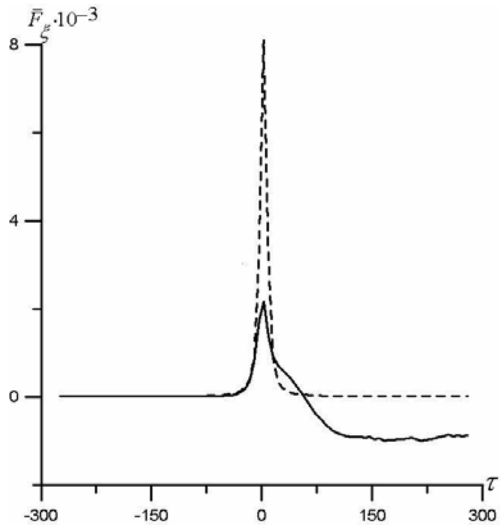

Fig. 4. The average effective force of the electrons interaction in units of the Coulomb energy at the wave-length vs. the dimensionless time (full line) (see Eqs. (18, 19)). The laser field intensity of the first and second waves are equal $I_1 = 0.78 \times 10^{15}$ and $I_2 = 1.4 \times 10^{15}$ W/cm^2. The pulse durations are $t_1 = 1.8 \times 10^{-13}$ s, $t_2 = 0.18 \times 10^{-13}$ s. The minimum distance between electrons is $\xi_{\min} = 2.26 \times 10^{-3}$ μm.

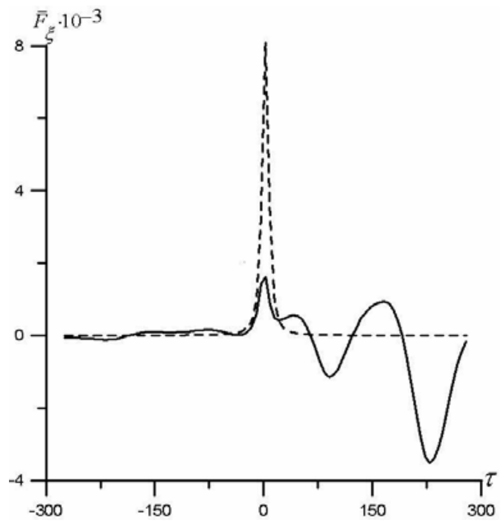

Fig. 5. The average effective force of the electrons interaction in units of the Coulomb energy at the wave-length vs. the dimensionless time (full line) (see Eqs. (18, 19)). The laser field intensity of the first and second waves are equal $I_1 = 0.78 \times 10^{15}$ and $I_2 = 1.4 \times 10^{15}$ W/cm^2. The pulse durations are equal $t_1 = t_2 = 1.8 \times 10^{-13}$ s. The wave frequencies are $\omega_1 = 3.3 \times 10^{15}$ s^{-1}, $\omega_2 = 3.18 \times 10^{15}$ s^{-1}.

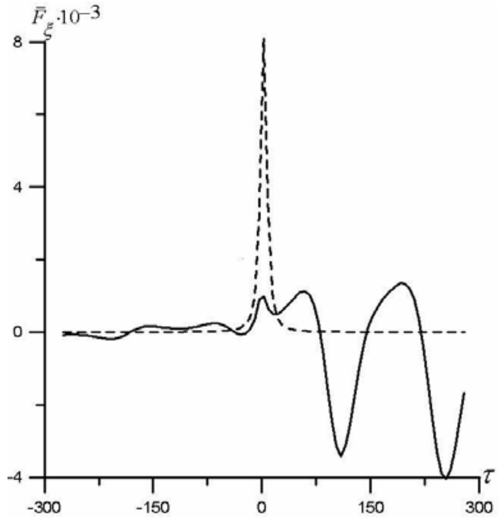

Fig. 6. The average effective force of the electrons interaction in units of the Coulomb energy at the wave-length vs. the dimensionless time (full line) (see Eqs. (18, 19)). The laser field intensity of the first and second waves are equal $I_1 = 0.78 \times 10^{15}$ and $I_2 = 1.4 \times 10^{15}$ W/cm^2. The pulse durations are equal $t_1 = t_2 = 1.8 \times 10^{-13}$ s. The wave frequencies are $\omega_1 = 3.3 \times 10^{15}$ s^{-1}, $\omega_2 = 3.64 \times 10^{15}$ s^{-1}.

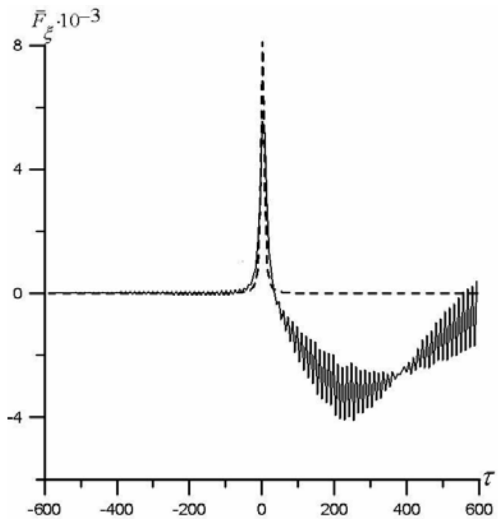

Fig. 7. The average effective force of the electrons interaction in units of the Coulomb energy at the wave-length vs. the dimensionless time (full line) (see Eqs. (18, 19)). The laser field intensity of the first and second waves are equal $I_1 = 0.3 \times 10^{15}$ and $I_2 = 0.78 \times 10^{15}$ W/cm^2. The pulse durations are equal $t_1 = t_2 = 1.8 \times 10^{-13}$ s. The wave frequencies are $\omega_1 = 3.03 \times 10^{15}$ s^{-1}, $\omega_2 = 12.1 \times 10^{15}$ s^{-1}. The minimum distance between electrons is $\xi_{\min} = 1.55 \times 10^{-3}$ μm.

At essential difference of frequencies there is an opportunity of a choice at which the area of negative values of average effective force of electrons interaction both extends and goes deep. Let's consider some examples. In Figures 7, 8 one can see, that there is a stable area of negative values of force. Let's note, that on size, average effective force of electrons interaction, is less than Coulomb interaction force (a dashed line in figure).

In Figure 9 one can see, that the area of negative values of the average effective force of the electrons interaction has changed. Size of the average effective force of the electrons interaction becomes essentially more then size of the Coulomb interaction force.

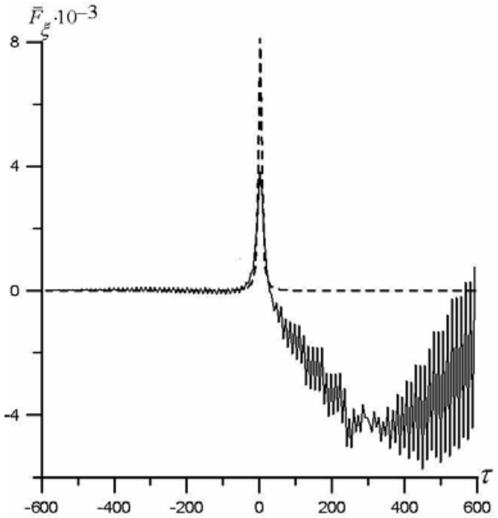

Fig. 8. The average effective force of the electrons interaction in units of the Coulomb energy at the wave-length vs. the dimensionless time (full line) (see Eqs. (18, 19)). The laser field intensity of the first and second waves are equal $I_1 = 0.3 \times 10^{15}$ and $I_2 = 2.16 \times 10^{15}\,\mathrm{W/cm^2}$. The pulse durations are equal $t_1 = t_2 = 1.8 \times 10^{-13}\,\mathrm{s}$. The wave frequencies are $\omega_1 = 3.03 \times 10^{15}\,\mathrm{s^{-1}}$, $\omega_2 = 12.1 \times 10^{15}\,\mathrm{s^{-1}}$. The minimum distance between electrons is $\xi_{\min} = 1.31 \times 10^{-3}\,\mu\mathrm{m}$.

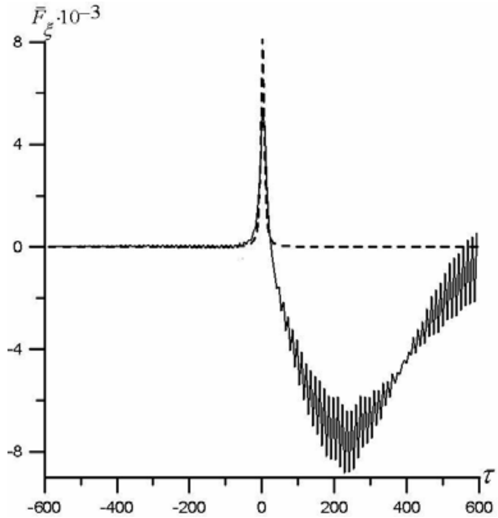

Fig. 9. The average effective force of the electrons interaction in units of the Coulomb energy at the wave-length vs. the dimensionless time (full line) (see Eqs. (18, 19)). The laser field intensity of the first and second waves are equal $I_1 = 0.3 \times 10^{15}$ and $I_2 = 2.16 \times 10^{15}\,\mathrm{W/cm^2}$. The pulse durations are $t_1 = 1.8 \times 10^{-13}\,\mathrm{s}$, $t_2 = 1.65 \times 10^{-13}\,\mathrm{s}$. The wave frequencies are $\omega_1 = 3.03 \times 10^{15}\,\mathrm{s^{-1}}$, $\omega_2 = 12.1 \times 10^{15}\,\mathrm{s^{-1}}$. The minimum distance between electrons is $\xi_{\min} = 1.31 \times 10^{-3}\,\mu\mathrm{m}$.

5 Conclusion

It is shown, that under certain conditions the average effective force of electrons interaction in the external electromagnetic field, created by two strong pulse laser waves, can have character of an attraction.

The novel results are next. The average effective force of the electrons interaction in presence of the two strong pulsed laser waves becomes attractive force under less wave intensities than in presence of one strong pulsed laser wave (by order of magnitude smaller).

The average effective force of the electrons interaction in presence of the two strong pulsed laser waves becomes attractive force in the next situations. The first one occurs when wave frequencies are equal. If the laser pulse durations are equal and the wave intensities are over the range $I = (0.35 \div 3.16) \times 10^{15}\,\mathrm{W/cm^2}$, then the average effective force has negative values during the time $\tau = 0.26 \times 10^{-13}\,\mathrm{s}$. The magnitude of force becomes smaller when the pulse durations are differ, but not too much. The second case takes place when wave frequencies differ considerably ($\omega_2/\omega_1 = 4$). The range of negative values of the average effective force expands in time duration and shift relative to laser pulse peak.

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